A FACTOR ANALYSIS STUDY OF THE MAJOR ELEMENT GEOCHEMISTRY OF GRANITIC ROCKS

RISTO PIISPANEN and TUOMO ALAPIETI

A Varimax-rotated R-mode factor analysis was performed on a collection of 195 whole-rock major element chemical analyses of granitic rocks, mainly Finnish. Factor loadings with corresponding eigenvalues and communalities were computed separately for 3 to 6 factors. The rapakivis, the non-rapakivis and the combined group of all the granitic rocks were each studied individually. The results show that, in contrast to the other granites, the rapakivis form a fairly homogeneous group with pronouncedly high mutual correlations between the components. A factor analysis, therefore, reveals one single factor with a very high eigenvalue. This factor is designated the femic factor on the basis of the most important participating components. In other granitic rocks three main factors emerge; these are termed the femic, the one-alkali and the biotite factor, respectively. The existence of the one-alkali factor reflects the even stronger negative correlation of Na_2O and K_2O in the non-rapakivis compared with the rapakivis.

Risto Piispanen and Tuomo Alapieti, Department of Geology, University of Oulu, SF-90100 Oulu 10, Finland.

Introduction

Since its early origins in psychology (see Koch and Link 1971, p. 121), factor analysis has been performed on collections of multivariate data within a variety of scientific fields. In geology applications of the method are already commonplace in palaeontology (*e.g.* Gould 1967), sedimentology (*e.g.* Krumbein and Graybill 1965; Imbrie and Purdy 1962), mineralogy (*e.g.* Saxena 1969) and economic geology (*e.g.* Koch and Link 1971), but in petrology and lithogeochemistry the applications to date are very few.

It is the aim of any factor analysis to try to find simple structure among a multitude of variables, and consequently to minimize the number of independent variables in terms of which the wealth of information is presented. A factor analysis thus creates new »artificial» properties which should cover to as great a degree as possible the total variances of the original properties. As a result of a successful factor analysis, for instance, information can be graphically displayed far more easily than would otherwise be possible, and the comprehensibility of the information contained in the data collection is thereby

increased. On the other hand, the new artificial variables created in association with the analysis are often difficult to interpret empirically in physical terms and their real nature and associated geological implications often prove to be interpretable only with difficulty.

When applying factor analysis to a collection of major element data of a rock-group the main interest is directed towards a possible parallelism of the mineralogical composition and the factor pattern. In a simple hypothetical system, in which each chemical component would have a mineralogical counterpart of its own, a high correlation between some of the components would induce a correspondingly high correlation among some of the mineral phases and, in the analysis therefore, a factor could be created which would represent both the chemical components and the minerals and could also be interpreted in terms of the mineral composition. In real rocks in which most of the chemical components occupy positions in more than one mineral's lattice and, furthermore, each mineral (with rare exceptions) comprises more than one chemical component, the situation is much more complicated. High correlations among the chemical components may exist but the factors that can be created on the basis of the correlations may not be easily interpretable in terms of the mineral composition. To see whether the mineral composition is reflected in the factor pattern is one of the interesting points of lithogeochemical factor analysis.

The number of factors that can be created in a factor analysis is dependent on the degree of correlation between the properties to be studied, in this case the chemical components. Similarly, the resulting eigenvalues and the communalities also reflect the degree of correlation and the successfulness of the analysis. In a closed system such as a whole-rock chemical analysis, i.e., in a system with a fixed sum (or nearly so) some apparent correlation resulting from arithmetical reasons can be of utmost importance. This type of correlation has been called induced correlation, and, Chayes in particular, has paid attention to it (Chayes 1962, Chayes and Kruskal 1966, Koch and Link 1971, p. 169). Therefore one of the interesting aspects of a factor analysis is also to study whether the induced correlation will manifest itself in the results of a lithogeochemical factor analysis.

Material, method and results

The collection of silicate analyses of granitic rocks that have been earlier described in detail by one of the authors (Piispanen 1977) was utilized as the basic data collection for the present study. For the data sources and other details the reader is referred to this earlier publication (Piispanen *op. cit.*). The collection consists of 195 analyses of which 40 represent rapakivis so that for the analyses three groups can be delimited, the total family of granitic rocks including the rapakivis, the rapakivis on their own and the total family excluding the rapakivis, the so-called non-rapakivi group.

The necessary computations were carried out on the University of Oulu computer system. The principle and the method of computation were based on the correlation matrix from which, as a first phase, a non-iterative principal axes solution was calculated. Each of the three subgroups of the data collection were thereafter individually subjected to a Varimax-rotated factor analysis in which pertinent loadings, eigenvalues and communalities were calculated by turns for 3 to 6 factors. The results of the relevant cases are shown in Table 1.

In Table 2 the results are summarized and converted into chemical components. For each individual factor the components listed are those which contribute most strongly to

Table 1. Factor loadings for individual runs covering 3 to 6 factors. A stands for the rapakivis, C for all granites (C = A + B) and B for the non-rapakivis (B = C — A).

	A			Commu-nality
	I	II	III	
SiO_2	—.759	—.268	—.566	.968
TiO_2	.826	.210	.247	.787
Al_2O_3	.190	—.023	.830	.726
Fe_2O_3	.499	.731	—.010	.784
FeO	.942	.030	.189	.924
MnO	.847	—.019	.190	.754
MgO	.554	.570	.318	.733
CaO	.842	.025	.375	.850
Na_2O	.314	—.662	.071	.543
K_2O	—.720	.313	.217	.663
P_2O_5	.609	.351	.463	.709
H_2O^+	—.086	.768	—.008	.597
H_2O^-	.173	.640	.180	.471
	5.190	2.637	1.683	9.509

	A				Commu-nality
	I	II	III	IV	
SiO_2	—.748	—.292	—.581	—.002	.982
TiO_2	—.795	.238	.245	.232	.803
Al_2O_3	.176	—.011	.833	.044	.728
Fe_2O_3	.492	.742	.000	—.092	.801
FeO	.945	.057	.209	—.007	.940
MnO	.829	.008	.194	.190	.761
MgO	.522	.590	.314	.140	.738
CaO	.862	.047	.408	—.188	.947
Na_2O	.300	—.648	.061	.307	.607
K_2O	—.720	.293	.209	—.135	.667
P_2O_5	.552	.378	.442	.401	.805
H_2O^+	—.115	.767	—.022	.062	.605
H_2O^-	.136	.648	.166	.152	.490
	5.007	2.704	1.709	.455	9.874

Table 1. cont.

	A					Commu-nality
	I	II	III	IV	V	
SiO_2	—.761	—.243	—.572	—.101	—.085	.982
TiO_2	.767	.161	.246	.396	.020	.832
Al_2O_5	.180	—.041	.831	.056	.046	.730
Fe_2O_3	.519	.725	.008	.088	—.002	.802
FeO	.948	.009	.189	.057	.097	.948
MnO	.822	—.065	.155	.091	.375	.852
MgO	.515	.533	.329	.353	—.028	.783
CaO	.883	.027	.399	—.060	—.065	.947
Na_2O	.246	—.701	.042	.208	.103	.607
K_2O	—.681	.337	.215	—.234	.084	.686
P_2O_5	.523	.279	.434	.449	.253	.806
H_2O^+	—.101	.754	—.004	.171	.043	.610
H_2O^-	.157	.605	.147	.068	.400	.577
	4.922	2.518	1.663	.651	.408	10.161

	A						Communality
	I	II	III	IV	V	VI	
SiO_2	—.751	—.269	—.569	—.151	—.061	—.003	.986
TiO_2	.746	.194	.240	.377	—.001	.197	.832
Al_2O_3	.177	—.023	.832	.084	.034	.006	.732
Fe_2O_3	.504	.698	—.014	.193	—.039	—.149	.802
FeO	.941	.041	.191	.089	.112	.056	.948
MnO	.798	—.029	.145	.167	.444	.055	.887
MgO	.481	.474	.281	.528	.012	—.104	.825
CaO	.887	.028	.397	.018	—.047	—.065	.952
Na_2O	.249	—.597	.080	—.004	.088	.434	.621
K_2O	—.683	.275	.192	—.093	.100	—.324	.703
P_2O_5	.489	.333	.427	.430	.216	.210	.807
H_2O^+	.112	.774	—.003	.123	—.089	.010	.635
$H_2O—$	.143	.688	.165	—.005	.271	.074	.599
	4.748	2.440	1.620	.733	.364	.426	10.330

Table 1. cont.

	B			Communality
	I	II	III	
SiO_2	—.820	.236	—.211	.772
TiO_2	.678	.062	.468	.683
Al_2O_3	.392	—.552	—.228	.511
Fe_2O_3	.231	.098	.718	.579
FeO	.802	.045	.073	.651
MnO	.102	—.085	.554	.325
MgO	.826	—.174	.162	.739
CaO	.674	—.488	—.055	.695
Na_2O	—.107	—.736	.264	.623
K_2O	—.356	.707	—.081	.634
P_2O_5	.481	—.026	.149	.255
H_2O^+	.664	.102	.138	.471
$H_2O—$	.121	.231	.095	.077
	3.956	1.758	1.300	7.014

	B				Communality
	I	II	III	IV	
SiO_2	—.550	.059	—.349	—.664	.868
TiO_2	.660	—.083	.469	.147	.684
Al_2O_3	.052	.175	—.055	.796	.670
Fe_2O_3	.226	—.083	.734	—.037	.598
FeO	.704	—.164	.112	.341	.651
MnO	.065	.061	.582	.051	.349
MgO	.769	.091	.164	.359	.755
CaO	.461	.238	.032	.661	.707
Na_2O	—.098	.733	.229	.188	.635
K_2O	—.387	—.707	—.012	—.300	.740
P_2O_5	.543	.077	.093	.047	.312
H_2O^+	.748	—.031	.068	.026	.566
$H_2O—$	.058	—.286	.146	.030	.107
	3.111	1.270	1.350	1.912	7.643

Table 1. cont.

	B						Communality
	I	II	III	IV	V		
SiO_2	—.515	.065	—.332	—.674	.217		.881
TiO_2	.643	—.085	.458	.162	—.178		.688
Al_2O_3	.067	.145	—.085	.823	.006		.709
Fe_2O_3	.278	—.129	.707	—.000	.086		.602
FeO	.502	—.060	.182	.256	—.617		.735
MnO	—.005	.079	.625	.018	—.131		.414
MgO	.668	.137	.189	.337	—.378		.757
CaO	.310	.302	.087	.609	—.416		.738
Na_2O	.002	.660	.204	.259	.343		.662
K_2O	—.297	—.746	—.066	—.281	.188		.763
P_2O_5	.602	.047	.051	.100	.019		.377
H_2O+	.757	—.030	.043	.055	—.145		.601
H_2O-	.092	—.307	.116	.046	.024		.119
	2.588	1.258	1.358	1.868	.976		8.047

	B						Communality
	I	II	III	IV	V	VI	
SiO_2	—.528	—.031	—.326	—.634	—.288	—.123	.886
TiO_2	.654	—.015	.450	.133	.172	.117	.691
Al_2O_3	.072	.214	—.086	.800	.142	—.054	.721
Fe_2O_3	.270	.063	.665	—.025	—.072	.293	.611
FeO	.524	—.202	.213	.203	.580	—.022	.739
MnO	.014	.066	.639	.009	.113	—.065	.431
MgO	.665	.102	.177	.257	.455	.014	.757
CaO	.274	.274	.041	.469	.627	.098	.775
Na_2O	—.035	.756	.143	.191	—.081	—.166	.663
K_2O	—.240	—.679	.002	—.123	—.482	.129	.782
P_2O_5	.581	.143	.009	.055	.054	.136	.383
H_2O+	.772	—.034	.051	.050	.118	—.037	.618
H_2O-	.055	—.126	.053	.000	.009	.496	.268
	2.572	1.250	1.272	1.444	1.344	.442	8.326

Table 1. cont.

	C			Communality
	I	II	III	
SiO_2	—.842	.138	—.263	.797
TiO_2	.655	.093	.519	.708
Al_2O_3	.449	—.494	—.239	.502
Fe_2O_3	.240	.105	.702	.561
FeO	.739	.157	.234	.625
MnO	.136	—.096	.576	.359
MgO	.740	—.247	.155	.632
CaO	.732	—.409	.008	.703
Na_2O	—.092	—.799	.159	.673
K_2O	—.329	.728	—.047	.641
P_2O_5	.521	—.021	.198	.311
H_2O+	563	.113	.135	.348
H_2O-	.173	.286	.143	.132
	3.780	1.808	1.403	6.991

Table 1 — *continued*

	C				Communality
	I	II	III	IV	
SiO_2	—.718	—.046	—.399	—.405	.841
TiO_2	.330	.153	.568	.503	.708
Al_2O_3	.672	—.306	—.119	.032	.560
Fe_2O_3	—.041	.065	.689	.285	.562
FeO	.566	.313	.367	.349	.675
MnO	.126	—.066	.630	—.022	.417
MgO	.446	—.177	.159	.664	.696
CaO	.777	—.207	.140	.280	.745
Na_2O	.095	—.809	.113	—.044	.679
K_2O	—.345	.684	—.020	—.305	.680
P_2O_5	.238	.008	.194	.504	.348
H_2O^+	.076	.075	.058	.762	.596
H_2O-	.018	.293	.164	.139	.132
	2.408	1.514	1.628	2.091	7.640

	C					Communality
	I	II	III	IV	V	
SiO_2	—.611	.061	—.388	—.447	—.386	.876
TiO_2	.119	—.076	.521	.507	.404	.712
Al_2O_3	.790	.183	—.069	.087	.032	.671
Fe_2O_3	—.011	—.133	.699	.305	—.022	.600
FeO	.168	—.107	.271	.336	.741	.775
MnO	.023	.102	.610	—.023	.205	.425
MgO	.319	.220	.141	.674	.268	.696
CaO	.579	.288	.109	.296	.489	.756
Na_2O	.280	.703	.179	—.030	—.285	.686
K_2O	—.260	—.742	—.022	—.296	—.124	.726
P_2O_5	.153	.012	.178	.514	.175	.349
H_2O^+	.024	—.069	.046	.768	.063	.602
H_2O-	.077	—.368	.175	.164	—.037	.200
	1.657	1.398	1.489	2.183	1.348	8.075

	C						Communality
	I	II	III	IV	V	VI	
SiO_2	—.610	.064	—.378	—.308	—.409	—.307	.876
TiO_2	.113	—.064	.491	.326	.418	.445	.736
Al_2O_3	.789	.182	—.077	.043	.041	.084	.673
Fe_2O_3	—.002	—.134	.723	.267	.009	.096	.621
FeO	.163	—.120	.254	.205	.752	.251	.776
MnO	.019	.107	.599	—.078	.214	.058	.426
MgO	.339	.194	.170	.631	.324	.169	.713
CaO	.588	.259	.123	.261	.527	.033	.775
Na_2O	.278	.722	.157	—.042	—.269	.058	.701
K_2O	—.286	—.721	—.040	—.322	—.176	.029	.738
P_2O_5	.139	.042	.124	.311	.171	.563	.479
H_2O^+	.043	—.085	.079	.725	.110	.235	.608
H_2O-	.084	—.375	.200	.163	—.027	.010	.215
	1.683	1.380	1.468	1.542	1.483	.781	8.338

Table 2. Summary of data from Table 1 converted into symbols for the most significant chemical components contributing to the generation of each individual factor. For the meaning of A, B, and C see Table 1. Cases from 3 to 6 factors are presented.

	A	B	C
I	—Si, Ti, Fe^{3+}, Fe^{2+}, Mn, Mg, Ca, —K,P	—Si, Ti, (Al), Fe^{2+}, Mg, Ca, P, H_2O^+	—Si, Ti, Al, Fe^{2+}, Mg, Ca, P, H_2O^+
II	Fe^{3+}, Mg, —Na, H_2O^+, H_2O^-	—Na, K, Al, (—Ca)	—Al, —Ca, —Na, K
III	—Si, Al, P	Ti, Fe^{3+}, Mn	Ti, Fe^{3+}, Mn
I	—Si, Ti, Fe^{3+}, Fe^{2+}, Mn, Mg, Ca, —K, P	—Si, Ti, Fe^{2+}, Mg, (Ca), P, H_2O^+	—Si, Al, Fe^{2+}, Mg, Ca
II	Fe^{3+}, Mg, —Na, H_2O^+, H_2O^-	Na, —K	—Na, K
III	—Si, Al, Ca, P	Ti, Fe^{3+}, Mn	Ti, Fe^{3+}, Mn
IV	(Na), P	—Si, Al, Ca	Ti, Mg, P, H_2O^+
I	—Si, Ti, Fe^{3+}, Fe^{2+}, Mn, Mg, Ca, —K, P	—Si, Ti, Fe^{2+}, Mg, P, H_2O^+	—Si, Al, Ca
II	Fe^{3+}, Mg, —Na, H_2O^+, H_2O^-	Na, K	Na, —K
III	—Si, Al, P, (Ca)	Ti, Fe^{3+}, Mn	Ti, Fe^{3+}, Mn
IV	P, (Ti), (Mg)	—Si, Al, Ca	Ti, Mg, P, H_2O^+
V	(Mn), H_2O^-	—Fe^{2+}, —Ca	Fe^{2+}, Ca
I	—Si, Ti, Fe^{3+}, Fe^{2+}, Mn, Mg, Ca, —K, P	—Si, Ti, Fe^{2+}, Mg, P, H_2O^+	—Si, Al, Ca
II	Fe^{3+}, (Mg), —Na, H_2O^+, H_2O^-	Na, —K	Na, —K
III	—Si, Al, (Ca), P	Ti, Fe^{3+}, Mn	Ti, Fe^{3+},Mn
IV	Mg, P, (Ti)	—Si, Al, Ca	Mg, H_2O^+
V	Mn	Fe^{2+}, Mg, Ca, —K	Fe^{2+}, Ca
VI	Na, (—K)	H_2O^-	P, (Ti)

the generation of an individual factor. The listed components regularly have loadings of 0.4 or higher.

Both rapakivis and the total group of all the granitic rocks have, as their first factor, a factor in which Fe^{2+}, Mg, Ca, Ti and negative silica have significant loadings. On the basis of the most important participating components the factor could, perhaps, be adequately called the femic factor. In rapakivis this factor has a very large eigenvalue (4.748 to 5.190) implying that a great deal of the variation of the chemical composition of the rapakivis can be described in terms of this single factor. The factor score pertai-

ning to this factor for any rapakivi would reveal a great deal about the relationship of that particular rapakivi to the others.

In summary, the factor analysis among the rapakivis reveals one clear and prominent factor and some additional less clear and less significant factors. This situation is obviously a result of the homogeneity of the rock-group in comparison with the heterogeneity of the total group of granites.

In addition to the first, the femic factor, the rapakivis exhibit a second factor in which many of the same components which formed the first factor have large loadings. The most significant exception is Na_2O which

here makes its first appearance, potassium being already present under the first factor. The allocation of soda and potassium to different factors in the rapakivis is discussed in association with the factors for the total family of the granitic rocks.

The third factor for the rapakivis is a negative silica-alumina factor. In this the arithmetically induced negative correlation between the two largest chemical components is obviously the reason for the formation of the factor.

In contrast with the rapakivis, the total group of granites have a second factor that carries large loadings for both potassium and soda. This factor is a result of the strong (-0.589) negative correlation of the components in the granites. Soda and potassium are also negatively correlated in the rapakivis, but the numerical value of the correlation (-0.521) there does not seem to be large enough to make the components fall under a common factor (in order to save space the correlation matrices are not reproduced here). A geological implication of these relationships is, perhaps, to indicate the ability of the rapakivi alkali feldspar to carry more soda either as perthite or antiperthite or as a homogeneous solid solution than that of the other granitic rocks.

As the third factor the total group of granites show, as a rule, a factor in which Ti, ferric iron and MnO have large loadings. This factor does not make its appearance among the rapakivis. The factor is most easily interpreted as a biotite factor containing the femic elements of biotite and excluding CaO which falls under the first factor, the femic factor, where hornblende is probably the main mineral representative.

REFERENCES

Chayes, F. (1962) Numerical correlation and petrographic variation. J. Geol. 70: 440—452.

Chayes, F. and *Kruskal, W.* (1966) An approximate statistical test for correlations between proportions. J. Geol. 74: 692—702.

Gould, S. J. (1967) Evolutionary patterns in Pelycosaurian reptiles: a factor analytic study. Evolution, 21: 385—401.

Imbrie, J. and *Purdy, E. G.* (1962) Classification of modern Bahamian carbonate sediments. AAPG Symposium on Classification of Carbonate Rocks, Memoir 1: 253—272.

Koch, G. S. and *Link, R. F.* (1971) Statistical Analysis of Geological Data. Vol. 2. J. Wiley & Sons. New York.

Krumbein, W. C. and *Graybill, F. A.* (1965) An Introduction to Statistical Models in Geology. McGraw-Hill. New York.

Piispanen, R. (1977) Major element geochemistry of the granitic rocks. Bull. Geol. Soc. Finland 49 (2): 73—78.

Saxena, S. K. (1969) Silicate solid solutions and geothermometry. 4. Statistical study of chemical data on garnets and clinopyroxene. Contr. Min. Petr. 23: 140—156.